Working towards and from the whole in division of fractions

Quotative (measurement) approach:

Example: Divide $\frac{1}{2}$ by $\frac{1}{3}$.

Traditional "into" thinking would ask: How many times does $\frac{1}{3}$ go into $\frac{1}{2}$, which works well with whole numbers. This seems to be a stumbling block for many children when trying to answer the same question with fractional quantities. It seems that their counting and fair-share strategies don't work anymore. However, "into" thinking works well with fractions if we first make common denominators: $\frac{1}{2} \div \frac{1}{3} = \frac{3}{6} \div \frac{2}{6} = 3 \div 2 = \frac{3}{2}$. Here we can think that two pieces of one-sixths go into three pieces of one-sixths one and a half times. By repartitioning (renaming) the fractions we have reduced the problem to a whole number problem, because we are comparing a count of equally sized pieces.

Moving toward multiplicative thinking (reciprocating):

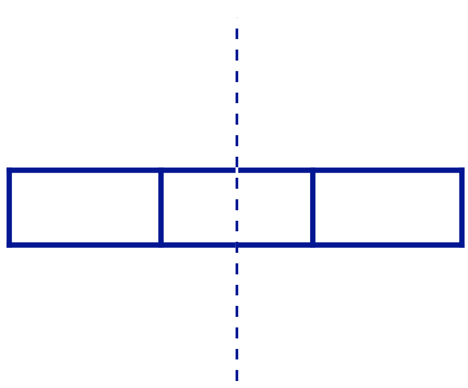
Example 1: $\frac{1}{2} \div \frac{1}{3}$

First think that you are going to find **how many pieces of $\frac{1}{3}$ make $\frac{1}{2}$**. (Context: How many $\frac{1}{3}$-foot ribbons do we need to make a $\frac{1}{2}$-foot ribbon?)

This we can do in steps:

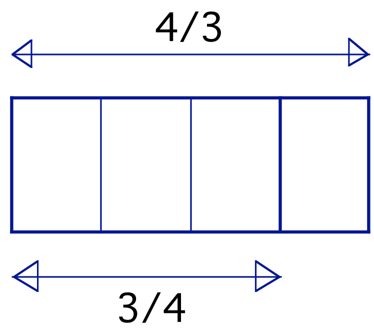
1. First find **how many pieces of $\frac{1}{3}$ make 1**. That will be 3 pieces.

2. Then consider, **How will I make $\frac{1}{2}$ from 1**? Make $\frac{1}{2}$ of 3 pieces: $3 \times \frac{1}{2} = 1\frac{1}{2}$

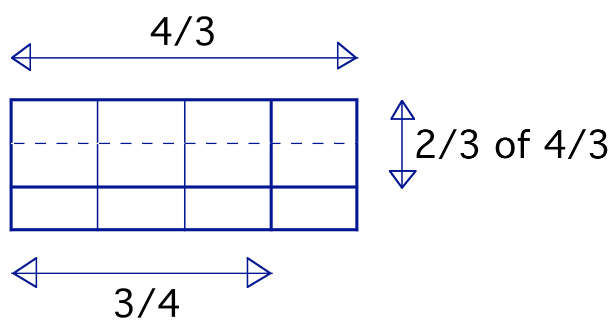


Example 2: $\frac{2}{3} \div \frac{3}{4}$

Step 1 Work toward the whole:
How many pieces of $\frac{3}{4}$ make 1? That will be $\frac{4}{3}$ pieces.



Step 2: Work from the whole:
How will I make $\frac{2}{3}$ from 1? Make $\frac{2}{3}$ of $\frac{4}{3}$ pieces: 4/3 x 2/3 = 8/9.



Algebraically this process can be summarized as follows:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times (1 \div \frac{c}{d}) = \frac{a}{b} \times \frac{d}{c}.$$

(See Liping Ma Chapter 4)